Practice Questions

Solutions

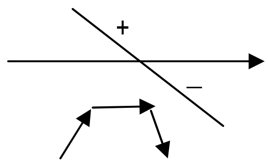
Curve Sketching

(a) (i) $\frac{d}{dx}\left(4x^{\frac{1}{4}}-x^{\frac{5}{4}}\right)=x^{-\frac{3}{4}}-\frac{5}{4}x^{\frac{1}{4}}=\frac{4-5x}{4x^{\frac{3}{4}}}$.

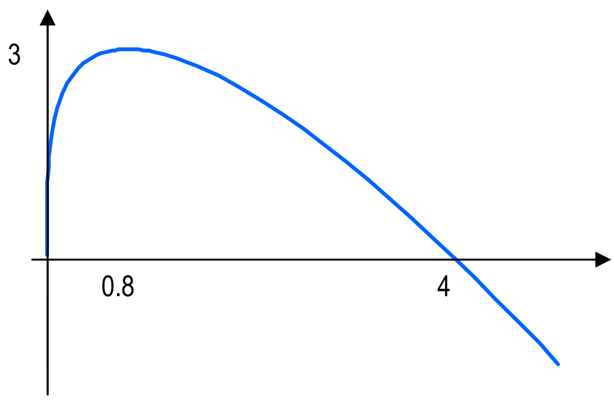
$\therefore$ As $x \to 0^{+}$, $f'(x) \to \infty$, the curve has a vertical tangent.

(ii) $f'(x)=0$ gives $x=\frac{4}{5}$. $\therefore$ Turning point $\left(\frac{4}{5},\frac{16}{5}\sqrt[4]{\frac{4}{5}}\right)\simeq\left(\frac{4}{5},3\right)$.

As $x^{\frac{3}{4}}>0$, the sign of $f'(x)$ is decided by $4-5x$, $\therefore$ this is a maximum point.



(iii) For the x-intercepts, let $y=0$, $x=0$ or 4.



(iv) Without using Calculus, sketch the following curves

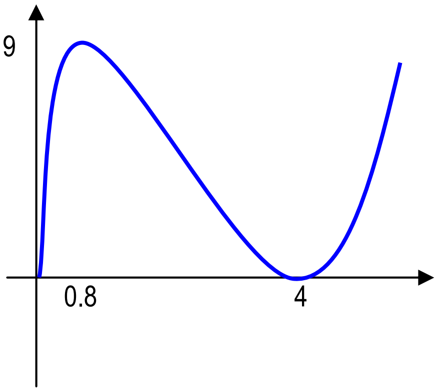
(α) $y = f^2(x)$.

(β) $y^2 = f(x)$.

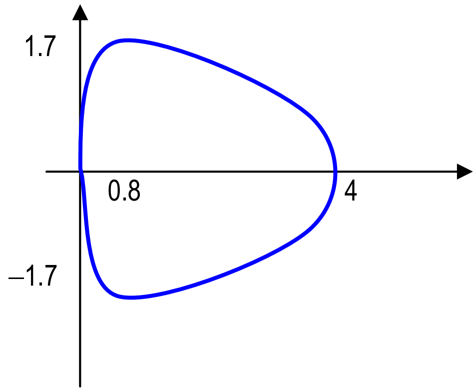
(γ) $y = \ln(f(x))$.

(δ) $y = e^{-f(x)}$.

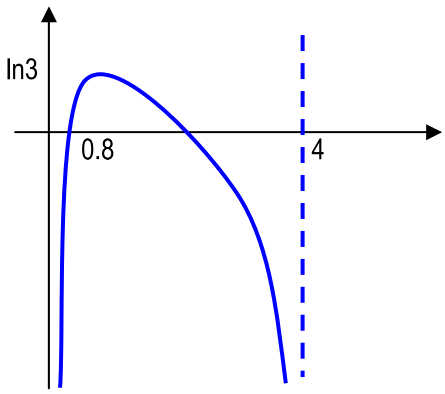
(α)



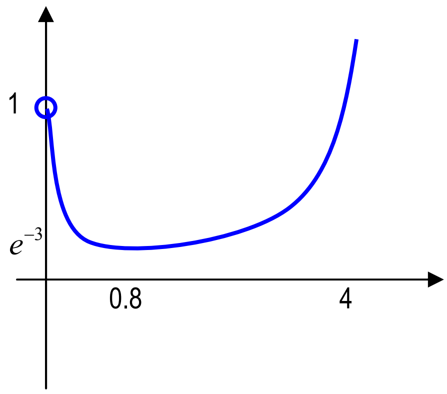
(β)



(γ)



(δ)



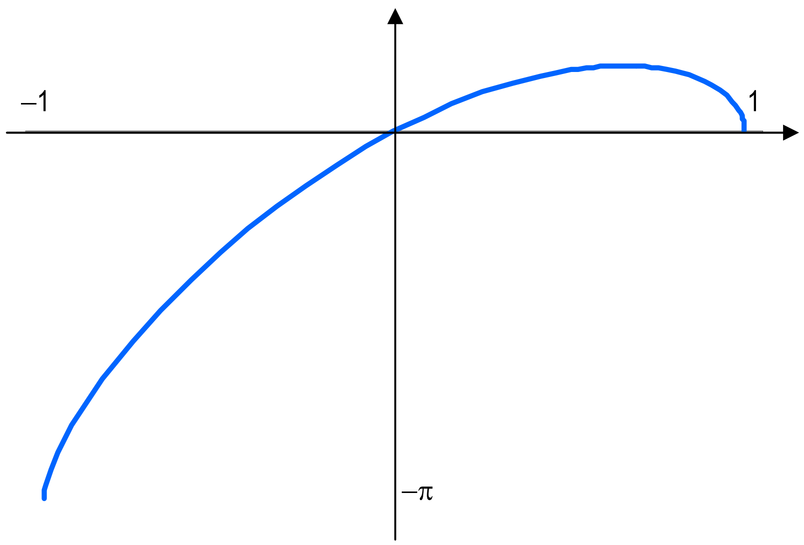
(b) (i) $f'(x) = \cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}}$

$$f''(x) = -\frac{1}{\sqrt{1-x^2}} - \frac{\sqrt{1-x^2} + \dfrac{x^2}{\sqrt{1-x^2}}}{1-x^2} = -\frac{1}{\sqrt{1-x^2}} - \frac{1-x^2+x^2}{\left(1-x^2\right)\sqrt{1-x^2}}$$

$$= -\frac{1-x^2}{\sqrt{\left(1-x^2\right)^3}} - \frac{1}{\sqrt{\left(1-x^2\right)^3}} = \frac{x^2-2}{\sqrt{\left(1-x^2\right)^3}}$$

(ii) For $-1 \le x \le 1, \sqrt{\left(1-x^2\right)^3} > 0$ but $x^2 - 2 < 0, \therefore f''(x) < 0$: the curve is concave downwards.

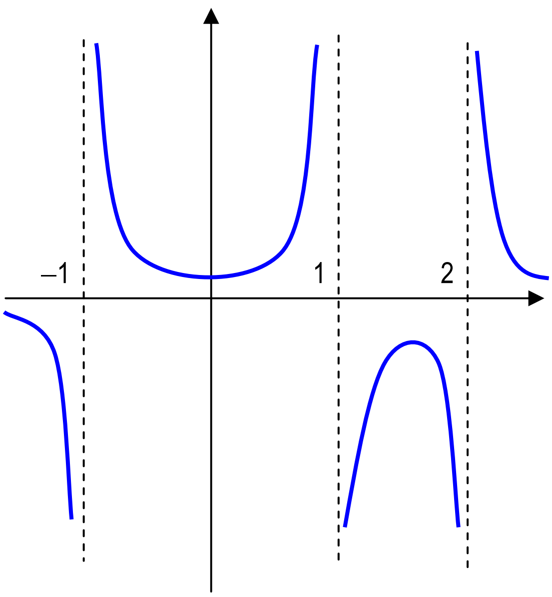
(iii)



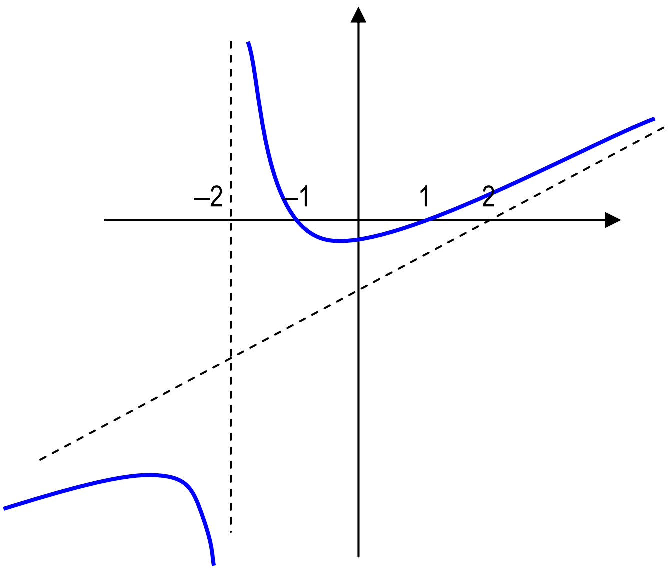
(c) (i) $y = \dfrac{1}{(x-2)(x^2-1)}$

(ii) $y = \dfrac{x^2-1}{x+2} = x - 2 + \dfrac{3}{x+2}$

(i)



(ii)

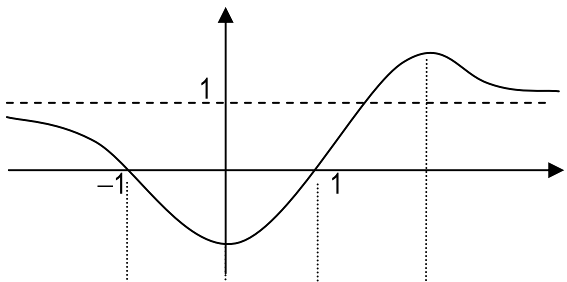


(d) Given the graph of $y = f'(x)$ as shown.

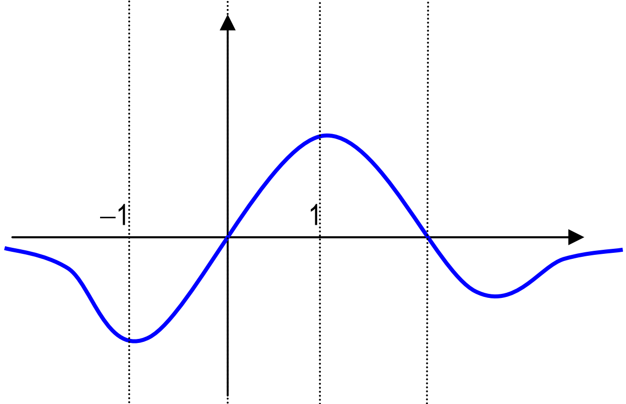
Sketch

(i) $y = f''(x)$.

(ii) $y = f(x)$, given $f(1) = 0$.



(i)



(ii) (The asymptote is $y = x + C$)