Chapter 1: Geometrical Applications of Calculus (15 marks)

(a) The curve $y = x^3 + x^2 - 5x + 1$ has two turning points A and B. (3 marks)

(i) What are the coordinates of A and B?

(ii) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions?

(b) Consider the curve $y = x^4 + 4x^3 - 16x$. (4 marks)

(i) Find and factorise y' completely and hence determine the location and nature of any stationary points.

(ii) Sketch the curve.

(c) Let $f(x) = \dfrac{2x}{x^2 - 1}$. (4 marks)

(i) For what values of x is $f(x)$ undefined? Also state the equation of the horizontal asymptote.

(ii) Show that $y = f(x)$ is an odd function.

(iii) Show that $f'(x) < 0$ at all values of x for which the function is defined.

(iv) Hence, sketch $y = f(x)$.

(d) Let $f(x) = 3x^5 - 10x^3 + 45x$. (4 marks)

(i) Show that $f'(x) > 0$ for all x.

(ii) For what values of x is $f''(x)$ positive?

(iii) Sketch the graph of $y = f(x)$.

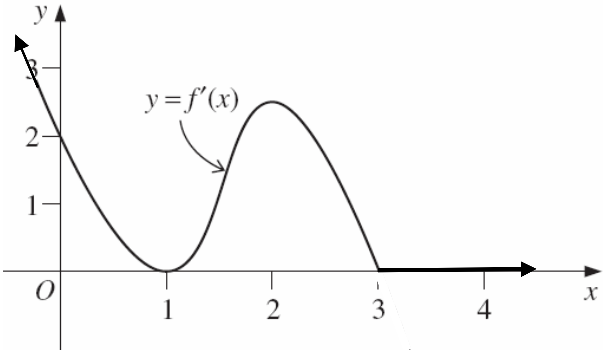
Chapter 2: Integration (25 marks)

(a) (i) Find the area of the region bounded by $y = x^2 - 4x, 2 \le x \le 5$ and the x-axis. (3 marks)

(ii) Find the volumes of the solids generated when the region enclosed by the curve $16y = x^2 + 16, -4 \le x \le 4$ and the line $y = 2$, is rotated about (α) the x-axis, (β) the y-axis. (6 marks)

(b) The diagram below shows a sketch of the gradient function $f'(x)$ of the curve $y = f(x)$.

Sketch the graphs of (α) $y = f''(x)$ and (β) $y = f(x)$ given that $f(0) = 0$. (4 marks)



(c) Evaluate the following integrals using the substitution given. (10 marks)

i) $\int \frac{x}{\sqrt{1+x^2}} dx$ using $u = 1 + x^2$.

ii) $\int x^3 \sqrt{x^2 - 4}\, dx$ using $u = x^2 - 4$.

iii) $\int_0^2 \frac{2x^3}{(x^2+1)^3} dx$ using $u = x^2 + 1$.

iv) $\int_{-1}^{0} 15x\sqrt{1+x}\, dx$ using $x = u^2 - 1$.

(d) Given that $\int_9^{10} \frac{1}{1+x} dx = \log_e 1.1$, explain why $1.1^{10} < e < 1.1^{11}$ by using a graphical means. (2 marks)

Chapter 3: Exponential and Logarithmic Functions (20 marks)

1) Solve $2^x = 3$, expressing your answer correct to 2 decimal places.

2) Differentiate with respect to *x*:

a) $y = \sqrt{e^{3-2x}}$

b) $y = \ln(4x-1)$,

c) $y = x\log_e x$,

d) $y = \log_{10}(5x-2)$

e) $y = \ln\dfrac{x-2}{x^2-3}$

f) $y = e^{\ln(x^5+6)}$

g) $y = 3^{x^5+4}$

h) $y = 3x^2\log_e x$

3) Find

a) $\int xe^{2x^2}\,dx$

b) $\int \dfrac{x}{3-x^2}\,dx$

c) $\int_0^{\ln 2} \dfrac{e^x}{e^x+1}\,dx$

d) $\int_0^2 \dfrac{x}{(x^2+1)^3}\,dx$

4) For the curve $y = xe^{2x}$

i) Find any stationary points

ii) Find any points of inflexion

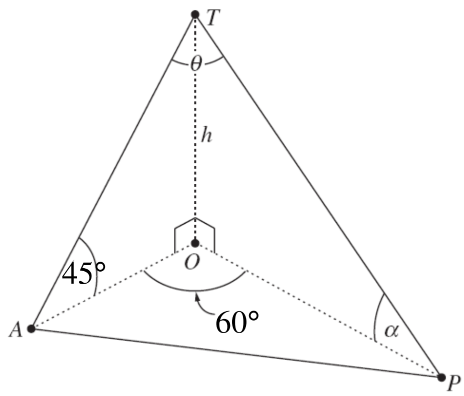
iii) Discuss the behaviour of the curve for positively and negatively large values of *x*

iv) Sketch the curve

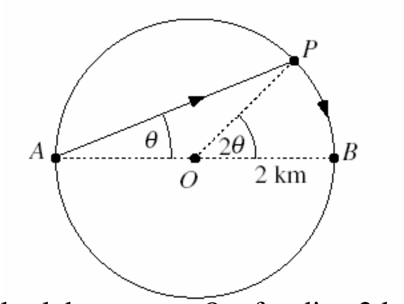
5) Find $\dfrac{d^2}{dx^2}\left(e^{x^3}\right)$

Chapter 4: Trigonometric Functions Test (52 marks)

1. (i) Prove the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. 3 marks
   (ii) Hence solve the equation $\sin 3\theta = 2\sin\theta$ for $0 \le \theta \le 2\pi$. 4 marks
2. Consider the diagram, which shows a vertical tower OT of height h metres, a fixed point A, and a variable point P that is constrained to move so that angle AOP is 60°. The angle of elevation of T from A is 45°. Let the angle of elevation of T from P be α radians and let angle ATP be θ°. 12 marks



   (i) By considering triangle AOP, show that $AP^2 = h^2 + h^2\cot^2\alpha - h^2\cot\alpha$.
   (ii) By finding a second expression for AP^2, deduce that $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$.
   (iii) Write $\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$ in the form $R\sin(\alpha+\beta)$, hence state the maximum value of $\cos\theta$.
3. Prove $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$ 3 marks
4. Solve the following equations, for $0 \le x \le 360°$ 13 marks
   (i) $\sin^2 x - 5\sin x - 2\cos^2 x = 0$ (ii) $\cot x - 3\tan x - 2 = 0$
   (iii) $\cos^2 x - 3\sin^2 x - \sin 2x = 0$ (iv) $1 + \cos 2x = \cos x$
5. Differentiate 3 marks
   a) $x^2 \sec 3x$ b) $\frac{\sin x}{1+\cos x}$
6. Evaluate 3 marks
   a) $\int x^2 \sin(x^3)\, dx$ b) $\int_{\frac{\pi}{8}}^{\frac{\pi}{3}} \sec^2 2x\, dx$
7. (i) Sketch the curves $y = 4\sin 2x$ and $y = x$ for $0 \le x \le \pi$ 5 marks
   (ii) Hence, solve $4\sin 2x \ge x$, for $0 \le x \le \pi$, correct to 1 d.p.
   (iii) Using the result of (ii) find the area enclosed by the curves $y = 4\sin 2x$ and $y = x$ for $0 \le x \le \pi$.
8. 6 marks



The diagram shows a circular lake, centre O, of radius 2 km with diameter AB. Pat can row at 3 km/h and can walk at 4 km/h and wishes to travel from A to B as quickly as possible. Pat considers the strategy of rowing direct from A to a point P and then walking around the edge of the lake to B.
Let $\angle PAB = \theta$ radians, and let the time taken for Pat to travel from A to B by this route be T hours.

(i) Show that T = $\frac{1}{3}(4\cos\theta + 3\theta)$.

(ii) Find the value of θ for which $\frac{dT}{d\theta} = 0$.

(iii) To what point P, if any, should Pat row to minimise T? Give reasons for your answer.