

Extension 1 – Test 2

1) Differentiate

(a) $y = x^2 \sin x$ (b) $y = \ln\left(\frac{2x+1}{x^2-1}\right)$ (c) $y = \frac{e^{3x}}{x^2}$ (d) $y = \frac{1}{\sqrt{1+4x^2}}$ 8

2) Find

(a) $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$ (b) $\int \frac{2x}{3x+4} dx$ (c) $\int \frac{\ln 2x}{x} dx$ (d) $\int \frac{1}{3+4x^2} dx$ 8

(e) $\int \frac{x}{\sqrt{1-x}} dx$ using $x = 1-u^2$ (f) $\int \frac{dx}{x^2\sqrt{1-x^2}}$ using the substitution $x = \cos\theta$ 6

3) Find $\frac{d^2}{dx^2}(e^{x^2})$. 4

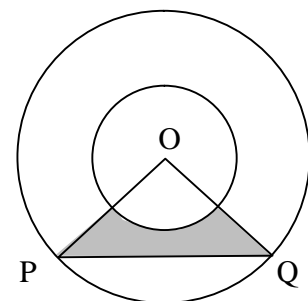
4) (a) Evaluate $\int_0^{\frac{\pi}{4}} \tan^2 x dx$. 3

(b) Evaluate $\int_0^1 x^3 \sqrt{x^2+1} dx$ using $u = x^2+1$. 4

(c) Use the substitution $u = e^x + 1$, evaluate $\int_0^1 \frac{e^{2x}}{e^x+1} dx$. 4

5) Differentiate $x \tan x$ with respect to x , hence, evaluate $\int_0^{\frac{\pi}{3}} x \sec^2 x dx$. 5

6) Two concentric circles have radii 2 cm and 4 cm and centre O. The points P and Q lie on the larger circle and $\angle POQ = x$, where $0 \leq x \leq \frac{\pi}{3}$.



(a) Prove that the area of the shaded region is $A = 8 \sin x - 2x$. 2

(b) Hence, find the value of x such that this area is maximum. 2

7) (a) If $y = x^2 e^{-x}$. Find the coordinates of any turning points and determine their nature. 4

(b) Sketch the curve. 3

8) (a) PQ, where P is $(2p, p^2)$ and Q is $(2q, q^2)$, is a focal chord of the parabola $4y = x^2$. Prove that $pq = -1$. 3

(b) Hence, find the length of PQ in terms of p in its simplest form. 3

9) (a) Sketch on the same axes the curves $y = \cos 2x$ and $y = -2 \cos^2 x$, for $0 \leq x \leq \pi$. 3

(b) Find the x -coordinates of the points of intersection, for $0 \leq x \leq \pi$. 3

(c) Find the area enclosed between the two curves, for $0 \leq x \leq \pi$. 5

10) (a) Sketch the curve $y = \tan^{-1} 2x$. 2

(b) Show that the area obtained by this curve, the x -axis and the line $y = \frac{\pi}{4}$ is $\frac{1}{4} \ln 2$. 3

11) (a) Write $\sqrt{3} \cos x - \sin x$ in the form $R \cos(x + \alpha)$, hence, find the maximum value of $\sqrt{3} \cos x - \sin x$. 3

(b) What is the maximum value of $(\sqrt{3} + 1) \cos x - \sin x$? 1

12) (a) Explain why $f(x) = x^2 + 2x + 5$ does not have an inverse function. 1

(b) Find the largest domain so that the restricted function may have an inverse function. 1

(c) Find the inverse function in the restricted domain. 2

13) Find the exact values of (Working must be shown)

(a) $\cos^{-1}\left(\cos \frac{5\pi}{4}\right)$ (b) $\sin\left(\tan^{-1} \frac{9}{40}\right)$ (c) $\sin\left(2 \cos^{-1} \frac{3}{5}\right)$ (d) $\sin^{-1} \frac{1}{2} - \sin^{-1} \frac{1}{3}$ 8

14) (a) If all the letters of the word ENTERTAINMENT are mixed up and arranged in a row, in how many ways will neither of the three letters T's be together? 2

(b) If only 5 letters are chosen, in how many ways does the selection contain more vowels than consonants? 3

15) Prove by mathematical induction that $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^n} > \frac{n}{2}$ for all n positive integers. 4